In a fluid queue model a divisible commodity (fluid) arrives at a storage facility where it is stored in a buffer and gradually released. In a standard queueing system we consider, individual customers or jobs arriving at service facility, possibly wait, then receive service and depart. For such models we count the number of customers in the system and describe the experience of individual customers. In contrast a fluid queue model is used in applications where individual customer is so small that they can hardly be distinguished. It is then easier to imagine a continuous stream of work that flows into the system instead of customers. Fluid queues are used to represent systems where some quantity accumulates or is depleted, gradually over time, subject to some random environment. Some examples of such systems are a dam or a reservoir in which water builds up due to rainfall, is temporarily stored and then released according to some release rule, communication networks where data carried by these networks are packaged in many small packets, etc .

For fluid queues models, we study the buffer content at any time t, which is the amount of work in the system, that can be of finite or infinite capacity. The buffer content cannot be negative, so when the buffer content decreases to zero, the buffer content stays zero as long as the net input rate is negative.

A fluid queue driven by a Markov process, is a two-dimensional Markov process, of which the first component, or level, varies according to the second component, the phase, which is the state of a Markov process evolving in the background. Fluid flows into the buffer at a rate which depends on the state of the background Markov process and fluid flows out from the buffer at some rate. In order that a stationary distribution for the buffer exist, the stationary net input rate should be negative.

We shall explain the concept of fluid queues driven by Markov process through few simple examples:

1. Consider a fluid queue driven by a single on-off source denoted by, {X(t), t $\geq 0\}$ which takes values from the state space $\{0, 1\}$. The sojourn time of the source spends in the on-off state are exponentially distributed with parameters λ and μ respectively. Consider an infinite capacity buffer whose content at time t is denoted by C(t). During the on period fluid builds up in the buffer at a constant rate $r_1 > 0$ and during the off period the fluid depletes

from the buffer at a constant rate $r_0 < 0$ as long as the buffer is non-empty. The two dimensional process {(X(t),C(t)), t$\geq$ 0} forms a Markov process. The buffer occupancy distribution is defined as

$$F_i(t,x) = P\{X(t) = i, C(t) \leq x\} \qquad , t \geq 0, x \geq 0, i \epsilon S$$

The following are the Kolmogorov equations satisfied by the Markov process:

$$\frac{\partial F_0(t,x)}{\partial t} + r_0 \frac{\partial F_0(t,x)}{\partial x} = -\lambda F_0(t,x) + \mu F_1(t,x)$$

$$\frac{\partial F_1(t,x)}{\partial t} + r_1 \frac{\partial F_1(t,x)}{\partial x} = \lambda F_0(t,x) - \mu F_1(t,x)$$

subject to the initial condition's,

$$F_0(0,x) = 1 \ , \ F_1(0,x) = 0$$

and the boundary condition's,

$$F_0(t,0) = q_0(t), F_1(t,0) = 0$$

where $q_0(t)$ is to be determined.

When the process is in equilibrium $\frac{\partial F_j(t,x)}{\partial t} \equiv 0$ and we let $F_j(t,x) \equiv F_j(x)$. Then the above system of equation reduces to

$$\begin{aligned} r_0 \frac{dF_0(t,x)}{dx} &= -\lambda F_0(x) + \mu F_1(x) \\ r_1 \frac{dF_1(x)}{dx} &= \lambda F_0(x) - \mu F_1(x) \end{aligned}$$

The solution to the above equation should satisfy

$\lim_{x \to \infty} F_i(x) = p_i$ for i = 0,1 where, p_i denotes the stationary probabilities of the background Markov process given by

$$p_0 = \frac{\mu}{\mu+\lambda}$$
$$p_1 = \frac{\lambda}{\mu+\lambda}$$

In order that a stationary distribution for C(t) exist, the stationary net input rate should be negative, that is,

$$p_0 r_0 + p_1 r_1 < 0$$

After putting the limiting condition and solving the resultant system of equations we get the steady state solution.

2. Now we consider a fluid queue driven by a finite state birth death process. Let $\{X(t)/ \ t \geq 0\}$ denote the background birth death process which takes values in S=$\{0,1,2,...,N\}$. Let λ_i and μ_i represent the birth and death rates respectively. When the system is in state i, the buffer content changes at a rate r_i(which can be both positive and negative).If the buffer is empty, and the Markov process is in a state i with $r_i < 0$, then the buffer remains empty.

We let

$$\pi_i = \Pi_{j=1}^{i-1} \frac{\lambda_j}{\mu_{j+1}} \qquad , i \epsilon S.$$

The stationary state probabilities p_i, of the background birth death process can then be represented as

$$p_i = \frac{\pi_i}{\sum_{j \epsilon S} \pi_j} \qquad , i \epsilon S.$$

In order that a limit distribution for C(t), the content of the reservoir at time t, exist, the stationary net input rate should be negative, that is,

$$\sum_{i=1}^{N} r_i p_i < 0$$

We assume that the above condition is satisfied. In what follows we let

S^{+}=$\{i\epsilon S|r_i > 0\}$ and S^{-}=$\{i\epsilon S|r_i < 0\}$.
$d^{+} = |S^{+}|$, $d^{-} = |S^{-}|$
Letting

$$F_i(t,x) = P\{X(t) = i, C(t) \leq x\} \qquad ,t \geq 0, x \geq 0, i\varepsilon S$$

The Kolmogorov equations for the two dimensional process {X(t), C(t)} are given by:

$$\frac{\partial F_i(t,x)}{\partial t} + r_i \frac{\partial F_i(t,x)}{\partial x} = -(\lambda_i + \mu_i)F_i(t,x) + \lambda_{i-1}F_{i-1}(t,x) + \mu_{i+1}F_{i+1}(t,x) \qquad ,i\epsilon S$$

But assuming that the process is in equilibrium, we may set $F_i(t,x)$=F_i(u) and $\frac{\partial F_i(t,x)}{\partial t}$=0 and, hence, obtain the system

$$\frac{dF_i(x)}{dx} = \frac{\lambda_{i-1}}{r}F_{i-1}(x) - \frac{\lambda_i + \mu_i}{r}F_i(x)\frac{\mu_{i+1}}{r}F_{i+1}(x),$$

satisfying the conditions
F_i(0)=0, $\quad ,i\epsilon S^{+}$ and
$lim_{u\to\infty}F_i(u) = p_i \qquad ,i\epsilon S$

The above equations are then solved to get the steady state solution.

References

[1] E.A. van Doorn and W.R.W Scheinhardt, Analysis of birth-death fluid queues, Proceedings of the Applied Mathematics Workshop, Korea Advanced Institute of Science and Technology, Taejon (1996), 13-29.

[2] Viswanathan Arunachalam, Vandana Gupta and S. Dharmaraja, A fluid queue modulated by two independent birth-death processes, Computers and Mathematics with Applications 60 (2010), 2433-2444.

[3] Bruno Sericola, Transient analysis of stochastic fluid models, Performance Evaluation 32 (1998), 245-263.