# Queuing Networks
# Modeling Virtual Laboratory

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# Outline

- Non Markovian Queues
- Performance Measures
- Simple Examples

# M/G/1 Queue

- Arrival follows Poisson process with rate \lambda
- Service time (B) has arbitrary distributions with given E(B) and E(B^2)
  - Service times are iid and independent of arrival times
  - E(service time) = 1/\mu
- Single server queue

# State Probabilites

$$X_{n+1} = \begin{cases} A_{n+1}, & X_n = 0 \\ X_n - 1 + A_{n+1}, & X_n \geq 1 \end{cases}$$

$$\begin{aligned} a_r &= P(A_n = r) \\ &= \int_0^\infty \frac{e^{-\lambda t}(\lambda t)^r}{r!} dG(t), \quad r = 0, 1, \ldots \end{aligned}$$

$$\begin{aligned} P_{ij} &= Prob\{X_{n+1} = j / X_n = i\} \\ &= \begin{cases} a_j, & j \geq 0 \\ a_{j-i+1}, & i \geq 1, j \geq i-1 \\ 0, & i \geq 1, j < i-1 \end{cases} \end{aligned}$$

$$P = [P_{ij}] = \begin{bmatrix} a_0 & a_1 & a_2 & . & . & . \\ a_0 & a_1 & a_2 & . & . & . \\ 0 & a_0 & a_1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{bmatrix}$$

$$v_j = \lim_{n \to \infty} P_{ij}(n), \quad j = 0, 1, 2, \ldots$$

# Pollaczek-Khinchin (P-K) mean formula

$$L_s = \rho + \frac{\lambda^2(E(B)^2)}{2(1-\rho)}.$$

$$L_q = L_s - \lambda E(B); \quad E(B) = \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W_s = \frac{L_s}{\lambda}$$

# Special Cases

- M/M/1 Queue

$$L_s = \rho + \frac{\rho^2}{2(1-\rho)}$$

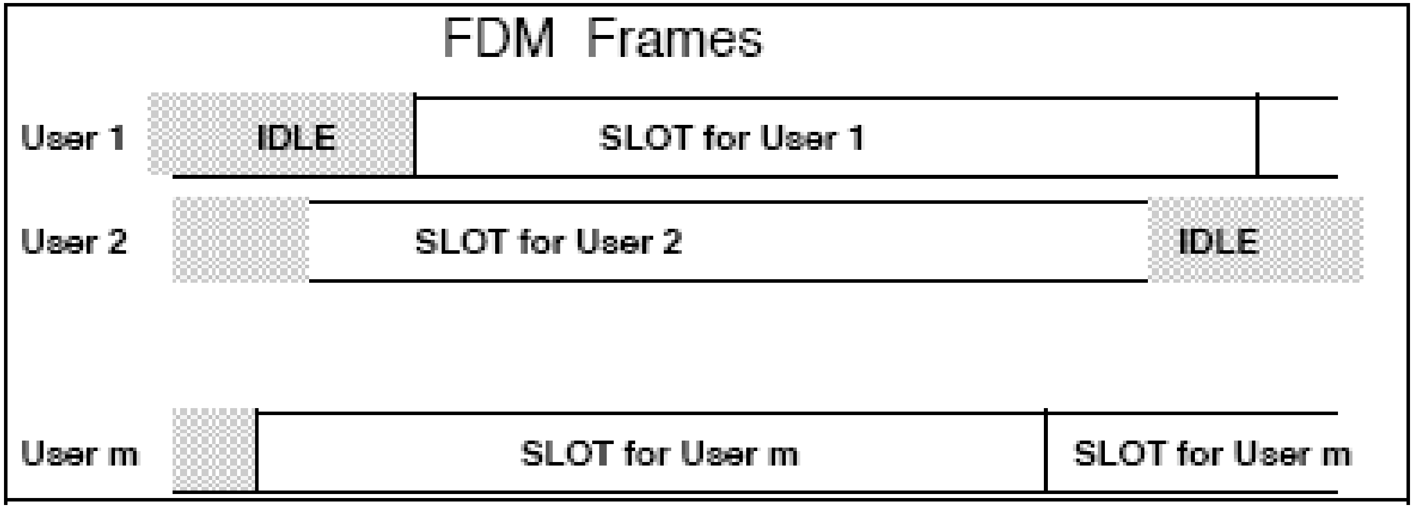
- M/D/1 Queue

$$\mathbf{E[X] = 1/\mu \ ; \quad E[X^2] = 1/\mu^2}$$

$$W = \frac{\lambda}{2\mu^2(1-\rho)} = \frac{\rho}{2\mu(1-\rho)}$$

# FDM Example

- Assume m Poisson streams (with rate \lambda/m) of fixed length packets each multiplexed by FDM on m subchannels. Total traffic = \lambda
- Suppose it takes m time units to transmit a packet, hence \mu = 1/m
- The total system load: \rho = \lambda



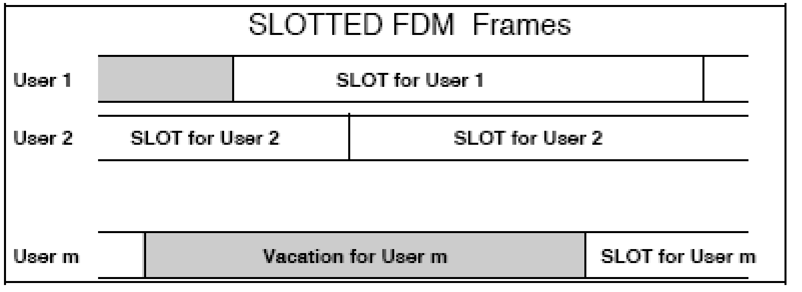
# FDM Example Cont…

By M/D/1 queueing system, we have

$$W_{FDM} = \frac{(\lambda / m)m^2}{2(1-\rho)} = \frac{\rho m}{2(1-\rho)}$$

# Slotted FDM Example

- Suppose that the system is slotted and transmissions start only m time unit boundaries
- Server goes on vacation for m time units where there is nothing to transmit
- E(V) = m,
- E(V^2) = m^2




$$W_{SFDM} = W_{FDM} + E[V^2]/2E[V]$$

$$= W_{FDM} + m/2$$