Open Queuing Network:

A queuing network refers to a system where there are several stations of service (identical or non-identical) and a customer undergoes service at all or few service stations. An open queuing network refers to a network in which the customers arriving to the system, leaves the system after completion of service. The experiments presented in the lab take into consideration a special class of open queuing networks called the Jackson networks. A queuing network is a Jackson network if it satisfies the following conditions:

- the network is open and any external arrivals to node i form a Poisson process.
- all service times are exponentially distributed and the service discipline at all queues is FCFS.
- a customer completing service at queue i will either move to some new queue j with probability P_{ij} or leave the system with probability $1-\sum_{j=1}^{m} P_{ij}$, which is non-zero for some subset of the queues.
- the utilization of all of the queues is less than one.

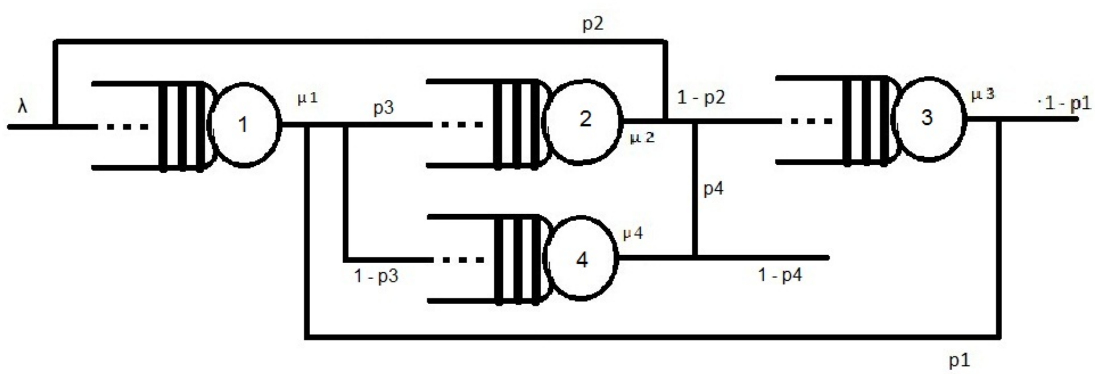
The steady state probability of the states of such networks is determined from the following theorem:

In an open Jackson network of m $M/M/1$ queues, where utilization ρ_i is less than 1, in every i^{th} queue, the steady state probability distribution exists and for state $\{k_1,\ldots,k_m\}$, it is given by:

$$\pi(k_1,\ldots,k_m) = \Pi_{i=1}^{m}\pi_i(k_i) = \Pi_{i=1}^{m}\rho_i^{k_i}(1-\rho_i)$$

The result also holds if a node is an $M/M/c$ queue with utilization of the node being less than c.

The network illustrated here is:



Each node in the network behaves as an $M/M/1$ queue. We shall determine the arrival rate at each node. Let λ be the arrival rate from an external source and λ_i denote the arrival rate at node i, $i = 1,2,3,4$. Then we have the following relations:

$$\lambda_1 = \lambda + p_2\lambda_2, \lambda_2 = p_3\lambda_1 + p_1\lambda_3, \lambda_3 = (1-p_2)\lambda_2 + p_4\lambda_4, \lambda_4 = (1-p_3)\lambda_1$$

Simplifying we obtain:

$$\lambda_1 = \frac{1-p_1(1-p_2)}{p_1p_4(1-p_3)-p_3}\lambda_2, \lambda_3 = \frac{\lambda_2}{p_1} - \frac{p_3}{p_1}\lambda_1, \lambda_4 = (1-p_3)\lambda_1$$

where

$$\lambda_2 = \frac{p_1p_4(1-p_3)-p_3}{1-p_1(1-p_2)-p_1p_2p_4(1-p_3)+p_2p_3}\lambda$$

Using the results of $M/M/1$ queue at each node i and Jackson's theorem, we obtain the desired results.