M/G/1 Queue

A memoryless, Poisson process always gives an exponential distribution for inter-arrival or inter-service times. However there are cases where the service process times are not exponentially distributed. In these cases, the service time distribution is said to be "general", and we describe the queue as M/G/1.

M/G/1/∞ Queue: Single server, Poisson arrival process with average arrival rate λ, Inter-arrival times exponentially distributed with mean $1/\lambda$, Service times Generally distributed with pdf *b(t)*, cdf *B(t)*, Infinite number of waiting positions and Service discipline assumed to be FCFS unless otherwise specified.

The process $\{N(t), t \geq 0\}$, where $N(t)$ gives the system size at time t, is non-Markovian. However, the analysis of such a process could be based on a Markovian process that can be extracted out of it. The two popular approaches that are used for this purpose are: Embedded Markov chain technique and Supplementary variable technique.

Measures of effectiveness:

1. Steady-state distribution of departure epoch system size:

$$V(s) = \frac{(1-\rho)(1-s)B^*(\lambda - \lambda s)}{B^*(\lambda - \lambda s) - s}$$

where $V(s)$ is the PGF of $\{v_j\}$, the limiting probabilities that a departing customer leaves behind j customers in the system; $B^*(s)$ is the LST of $B(t)$. This formula is known as Pollaczek-Khinchin (P-K) formula.

2. PGF $Q(s)$ of the number of customers in the queue is:

$$Q(s) = \frac{(1-\rho)(1-s)}{B^*(\lambda - \lambda s) - s}$$

3. Expected waiting time in the queue:

$$E(W_q) = \frac{\rho}{2\mu(1-\rho)}(1 + {c_v}^2)$$

where c_v is the coefficient of variation of service time v and $1/\mu$ is the mean service time.

4. Expected waiting time in the system:

$$E(W) = \frac{1}{\mu} + \frac{\lambda}{2(1-\rho)} E(v^2)$$

5. Expected number in the system:

$$E(N) = \rho + \frac{\rho^2}{1-\rho}(1 + c_v{}^2)$$

6. Expected number in the queue:

$$E(L_q) = \frac{\lambda^2}{2(1-\rho)} E(v^2)$$