DISCRETE TIME QUEUES

In a general scenario we have seen that the inter-arrival and inter-service times follow a continuous distribution or are deterministic in nature. But to the complicated and irregular service mechanisms in telecommunication networks, nowadays, required an approach whereby the time intervals are discrete in nature or the inter-arrival and service times follow a discrete probability distribution such as geometric distribution (for arrival of say two types of customers); discrete uniform distribution (where we have a closed system with N identical components) etc.. In this part of the lab we shall explore discrete-time models with non-deterministic service times. The inter-arrival times and the service times are discrete random variables taking on positive or non-negative integral values.

BULK QUEUES

This is a class of queues in which arrival or service (or both) is in bulks. This scenario is commonly found in ticket booking counters where a group of people arrive for ticket booking but they are served one at a time. This is referred to as BULK ARRIVAL. Another form of it is commonly seen in tourist places where a group of people arrive and a single tourist guide takes them through the place. This is referred to as BULK SERVICE, since one server, the tourist guide servers a group of people at the same time.

Bulk Arrival:
This form of the queue is an extension of the $M/M/1$ queuing system whereby we now make an additional assumption that x ($x \geq 1$) customers arrive, with probability c_x, for service and are served one at a time by the server (assuming 1 server). It is denoted by: $M^{[X]}/M/1$. If λ_x is the arrival rate of a batch of size X, then $c_x = \frac{\lambda_x}{\lambda}$, λ being the composite arrival rate of all the batches and equals $\sum_{x=1}^{\infty} \lambda_x$. This is also known as compound Poisson process. We can similarly extend $M/M/c, M/M/c/c, M/M/c/k$ models in terms of bulk arrivals.

For an $M^{[X]}/M/1$ system, the measures of effectiveness are

a) Average number of customers in the system:

$$L = \frac{r(E(X)+E(X^2))}{2(1-\rho)} = \frac{\rho + rE(X^2)}{2(1-\rho)}, \left(\rho = \frac{\lambda E(X)}{\mu}, r = \frac{\lambda}{\mu}\right)$$

b) Average number of customers in the queue: By Little's formula, we get:

$$L_q = L - \rho = \frac{\rho^2 + (r-1)\rho}{2(1-\rho)} \text{ for a batch of size } \left(\rho = \frac{\lambda E(X)}{\mu}, r = \frac{\lambda}{\mu}\right)$$

c) Steady State Probability: The probability of \$n\$ customers in the system is given by:

$$p_n = (1-\rho)\{[\alpha + (1-\alpha)\rho]^{n-1}[(1-\alpha)\rho]\}$$

Bulk Service:
In this form of queuing system, the customers arrive to the server one at a time and the server is capable of serving K customers at a time. This shows two possibilities: (a) the server waits to serve till there are K customers to be served or (b) the server performs service if there are $0 < i \leq K$ customers awaiting service. Hence, depending on the system, anyone of the disciplines is applicable. We denote a single server, bulk service by $M/M^{[X]}/1$. The other queuing models can be similarly extended.

It should be noted that the Markovian nature in these forms of queuing systems is maintained since the inter-arrival and service times are assumed to follow exponential distribution.

RETRIAL QUEUES

It often happens that a telecaller when dialing a number gets a busy signal, then the caller repeats the call after sometime. There may be many such callers. Such callers become a source of repeated calls and remain in what is termed as an *Orbit*. This kind of queueing system is referred to as the *retrial queueing system*. 'Retrial' is a queueing discipline which is observed in many queueing systems. Such as $M/M/1$ with retrial, $M/M/C$ with retrial and so on.