Pre-experiment and Post-experiment Quizzes

1. [1] Discuss the following queueing situations in terms of the characteristics of a queueing process:
   (a) Aircraft landing at an airport.
   (b) Supermarket checkout procedures.
   (c) Post office or bank customer windows.
   (d) Toll booths on a bridge or highway.
   (e) Processing of programs coming from a number of independent sources on a local area network into a central computer.

2. [1] A graduate research assistant "moonlights" in the food court in the students union in the evenings. He is the only one on duty at the counter during the hours he works. Arrivals to the counter seem to follow the Poisson distribution with mean of 10/hour. Each customer is served only one at a time and the service time is thought to follow an exponential distribution with a mean of 4 minute. Answer the following questions:
   (a) What is the probability of having a queue?
   (b) What is the average queue length?
   (c) What is the average time a customer spends in the system?
   (d) What is the probability of a customer spending more than 5 minutes in the queue before being waited on?
   (e) The graduate assistant would like to spend his idle time grading papers. If he can grade 22 papers an hour on average when working continuously, how many papers per hour can he average while working his shift.

3. [2] Traffic to a message switching center for Extraterrestrial Communication Corporation arrives in a random pattern (remember that 'random pattern' means exponential inter-arrival time) at an average rate of 240 messages/min. The line has the transmission rate of 800 characters/second. The message length distribution (including control characters) is approximately exponential with an average length of 176 characters. Calculate the principal statistical measures of system performance assuming that a very large number of message buffers is provided. What is the probability that 10 or more messages are waiting to be transmitted? What would be the average response time if the traffic rate into the center increased by 10%? Suppose however that it is decided to provide only the minimum number of message buffers required to guarantee that $p_k < 0.005$, the how many buffers should be provided? For this number of buffers calculate L, L_q, W, W_q. What is the probability that the time an arriving message spends in the system does not exceed 2.5 sec? What is the probability that the queuing time of a message before transmission is begun does not exceed 2.5 seconds?

4. [2] The Sad Sack Clothing Co. has decided to install a tie-line telephone system between its east coast and west coast facilities. A caller receives a busy signal if the call is dialed when all the lines are in use. An average of 105 calls/hour with an average length of 4 minutes is expected. Enough lines are to be provided to ensure that the probability of getting a busy signal will not exceed 0.005. How many lines should be provided? How many lines are required if the probability of a busy signal is not to exceed 0.01? What would the performance be with 10 lines?

5. [2] High Tale Airfreight has 20 buffered terminals on one communication line. The terminals are used for data entry to a computer system. The average time required to key an entry into the buffer is 80 seconds; this keying time is approximately exponential. High Tale analysts have found that they can model such systems using the machine repair model with average service time equal to 2 seconds. Calculate the throughput and the mean response time. Repeat the calculations if 50 terminals are put on the line but everything else remain the same.

6. [2] People arrive at a telephone booth at a Fly-by-Night airline terminal in a random pattern with an average inter-arrival time of 12 minutes. The length of phone calls from the booth, including the dialing time, wrong numbers etc. is exponentially distributed with an average time of 4 minutes.
   (a) What is the probability that an arriving person will have to wait?
   (b) What is the average length of the waiting lines that form from time to time; i.e., those that are not of zero length?
   (c) What is the probability that an arrival will have to wait for more than 10 minutes before the phone is available?
   (d) The telephone company plans to add a second booth when the traffic increases so much that W_q >= 5 minutes. At what average arrival time will W_q = 5 minutes occur?

7. [2] A clerk provided exponentially distributed service time to customers who arrive randomly at the average rate of 15/hour. What average service time must the clerk provide in order that 90% of all customers will queue for service for a time not exceeding 12 minutes?

8. [2] A Hopeless Junction a small full service gas station is operated by the owner. On Monday mornings customers arrive randomly at the average rate of 15/hour. The owner provides exponential service with a mean service time of 2.5 minutes. Answer the following:
   (a) What is the mean number of customers waiting for service?
   (b) What is the mean queuing time in minutes?
   (c) What is the mean time a customer spends at the station?
   (d) What is the mean number of customers at the station?
   (e) What is the probability that the owner is idle?
   (f) What fraction of time does the owner have customers waiting?
   (g) What is the mean number of customers waiting for service when one or more are waiting for service? Compare the answer with part (a).

9. [2] The manager of the Information center at Gritty Soap provides 3 consultants to help personal computer users solve their problems. PC users with problems arrive randomly at an average rate of 20 per 8 hour day. The amount of time that a consultant spends with a PC user has an exponential distribution with average value of 40 minutes. Users are signed to consultants in the order of their arrival.
   (a) What fraction of time is each consultant busy?
   (b) What is the mean time a user spends in the queue?
   (c) What is the mean number of users waiting for a consultant?
   (d) What is the mean time a user spends in the Information Center?
   (e) What is the mean number of users in the Center?
   (f) What is the probability that all the consultants are idle?
   (g) What is the probability that all the consultants are busy but no one is waiting in the line?

10. [3] The US Postal Service has announced that no customer will have to wait more than 5 minutes in line in a post office. It wants to know how many service windows should be kept open to keep this promise. You are hired as a consultant to help decide this. You realize that this is an impossible promise to keep, since the service times and arrival rates of customers are beyond the control of the Post Office. Hence you decide to concentrate on the average wait in the queue. Thus you want to decide how many service windows to keep so that the mean queueing time (excluding service time) is less than 5 minutes.

11. [3] Packets arrive at an infinite capacity buffer according to a Poisson process with a rate of 400 per second. All packets are exactly 512 bytes long. The buffer is emptied at a rate of 2 megabits per second. Compute the expected amount of time a packet waits in the buffer before transmission.

References:

1. Donald Gross and Carl M. Harris, "Fundamentals of Queueing Theory", John Wiley and Sons, 1998.

2. Arnold O. Allen, "Probability, Statistics, and Queueing Theory with Computer Science Applications", Acedemic Press, San Diego, 1977.

3. V. G Kulkarni, "Modeling, Analysis, Design, and Control of Stochastic Systems", Springer-Verlag, New York, 1999.